The Serial Correlation of Stock Market Realized Volatility

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Introduction

This paper extends the literature examining realized volatility. Previous studies suggest stock market volatility history has a positive serial correlation (for example, Engle (1982), Bollserslev (1986), and Bollerslev et al. (1992a)). We use Kalman Filter to estimate the time varying serial correlation of realized volatility. Our results suggest minimal or negative serial correlations during low volatility periods and positive spikes during extreme price movement. Such asymmetric market reactions help to clarify the price dynamics of U.S. equity markets.

Data and Analysis

For this analysis, we use the daily return data on the U.S. stock market index constructed and updated by Professor Kenneth R. French from July 1st, 1926 through May 31st, 2022. Following Jones and Wilson (1989), we calculate monthly realized volatility as the standard deviation of daily log-returns from each monthly. Denote s_t is the monthly volatility measure and s_{t-1} is this measure lagged by one month, we model the dynamic volatility series as follows.

$$s_t = a_t + b_t s_{t-1} + \varepsilon_t^s$$
$$\begin{bmatrix} a_t \\ b_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^b \end{bmatrix}$$

where a_t is the intercept, b_t reflects the serial correlation, and the ε_t^s is an error term with constant variance. We use the Kalman Filter framework to model a dynamic transition of the coefficient, in which ε_t^a and ε_t^b are the error terms for a_t and b_t. Figure 1 shows the time varying intercept and serial correlation.

Figure 1 – Time varying coefficients from the Kalman filter estimation

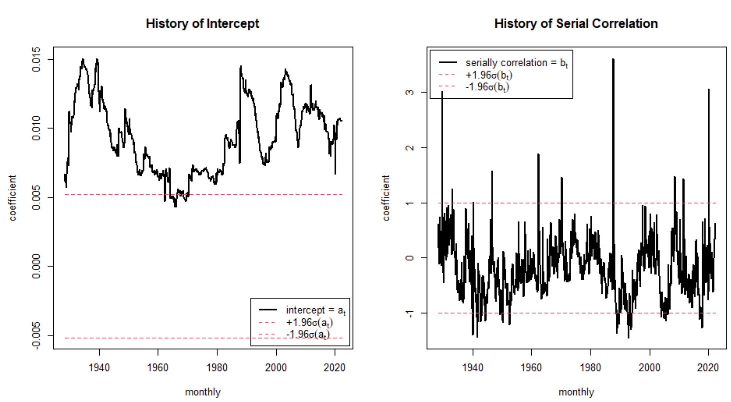
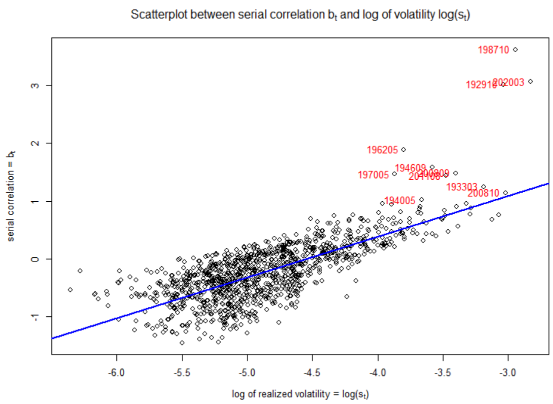


Figure 2 depicts the relation between serial correlation over log-volatility. Further regression analysis confirms a positive association between the serial correlation over the prior period volatility and current volatility. However, the serial correlation can be either negative or positive, which suggests asymmetric market reaction to volatility. When volatility is low during calm periods, the serial correlation tends to be negative or minimal, which provides a damping effect on the volatility. While during periods of extreme volatility, the serial correlation rises sharply into positive area, which contributes to the volatility spikes during periods of market panic.

Figure 2: Scatterplot of Serial Correlation over Log-volatility



Conclusion

This study uses Kalman Filter to conduct a dynamic regression of stock market realized volatility. We provide empirical evidence and rational for the investors' asymmetric behavior over periods with low vs. high volatility. In addition, this paper notes that there exists a positive consistent intercept term in volatility. This incept may lead to the previous conclusion that volatility is positively serial correlated, which misses the asymmetric behavior of serial correlation of volatility over time as highlighted in our study.

Further research in this area may wish to examine this asymmetric behavior of investors and the reasons why investors behave as they do during low volatility and high volatility times.

Reference

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